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Application of the Cercignani–Lampis scattering kernel to calculations of rarefied gas flows. I. Plane flow between two parallel plates

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Abstract

The Cercignani–Lampis scattering kernel of the gas-surface interaction was applied to numerical calculations of the plane Poiseuille flow, thermal creep, mechanocaloric flux and heat flux. The S model of the Boltzmann equation was numerically solved by the discrete velocity method. The calculations have been carried out in wide ranges of the rarefaction parameter and of the accommodation coefficients of momentum and energy. Comparing the present results with experimental data the value of the accommodation coefficients can be calculated. © 2002 Éditions scientifiques et médicales Elsevier SAS. All rights reserved.

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1. Introduction

Numerical calculations of rarefied gas flows are based on the Boltzmann equation, which can be expressed as [1]

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}'} = Q(ff_*). \tag{1}$$

Here, $f(t, \mathbf{r}', \mathbf{v})$ is the distribution function, t is the time, $\mathbf{r}'$ is the position vector, $\mathbf{v}$ is the molecular velocity, $Q(ff_*)$ is the collision integral. On the solid surface restricting the gas flow the distribution function satisfies a boundary condition, which in general form is expressed via a scattering kernel $R(\mathbf{v}' \to \mathbf{v})$ as [1]

$$|v_n|\, f(\mathbf{v}) = \int\limits_{v_n'<0} |v_n'| R(\mathbf{v}' \to \mathbf{v}) f(\mathbf{v}')\, \mathrm{d}\mathbf{v}', \tag{2}$$

where $\mathbf{v}'$ and $\mathbf{v}$ are molecular velocities of the incident and reflected particles, respectively, v_n is the normal component of the velocity $\mathbf{v}$ directed into the gas. The scattering kernel obeys the normalization condition

$$\int\limits_{v_n>0} R(\mathbf{v}' \to \mathbf{v})\, \mathrm{d}\mathbf{v} = 1, \tag{3}$$

and the reciprocity relation

$$|v_n'| \exp\left(-\frac{mv'^2}{2kT_w}\right) R(\mathbf{v}' \to \mathbf{v}) = |v_n| \exp\left(-\frac{mv^2}{2kT_w}\right) R(-\mathbf{v} \to -\mathbf{v}'), \tag{4}$$

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where m is the molecular mass of gaseous particles, k is the Boltzmann constant and T_w is the surface temperature.

In case of diffuse scattering the kernel becomes

$$R_d(\mathbf{v}' \to \mathbf{v}) = \frac{m^2 v_n}{2\pi (kT_w)^2} \exp\left(-\frac{mv^2}{2kT_w}\right). \tag{5}$$

It is easily verified that substituting this kernel into (2) the distribution function of reflected particles $f(\mathbf{v})$ becomes the Maxwellian.

The gas surface interaction can be described also using the accommodation coefficients defined as

$$\alpha(\varphi) = \frac{\int_{v_n'<0} \varphi(\mathbf{v}')|v_n'| f(\mathbf{v}')\,\mathrm{d}\mathbf{v}' - \int_{v_n>0} \varphi(\mathbf{v})|v_n| f(\mathbf{v})\,\mathrm{d}\mathbf{v}}{\int_{v_n'<0} \varphi(\mathbf{v}')|v_n'| f(\mathbf{v}')\,\mathrm{d}\mathbf{v}' - \int_{v_n>0} \varphi(\mathbf{v})|v_n| f_d(\mathbf{v})\,\mathrm{d}\mathbf{v}}, \tag{6}$$

where $\varphi(\mathbf{v})$ is some given function of the molecular velocity, $f(\mathbf{v})$ is the distribution function of reflected particles calculated for an arbitrary scattering kernel and $f_d(\mathbf{v})$ is calculated for the diffuse scattering kernel (6). Substituting (2) into (6) we obtain

$$\alpha(\varphi) = \frac{\int_{v_n'<0} \varphi(\mathbf{v}')|v_n'| f(\mathbf{v}')\,\mathrm{d}\mathbf{v}' - \int_{v_n'<0} |v_n'| f(\mathbf{v}') \int_{v_n>0} \varphi(\mathbf{v}) R(\mathbf{v}' \to \mathbf{v})\,\mathrm{d}\mathbf{v}\,\mathrm{d}\mathbf{v}'}{\int_{v_n'<0} \varphi(\mathbf{v}')|v_n'| f(\mathbf{v}')\,\mathrm{d}\mathbf{v}' - \int_{v_n'<0} |v_n'| f(\mathbf{v}') \int_{v_n>0} \varphi(\mathbf{v}) R_d(\mathbf{v}' \to \mathbf{v})\,\mathrm{d}\mathbf{v}\,\mathrm{d}\mathbf{v}'}, \tag{7}$$

where the order of the integration with respect to $\mathbf{v}$ and $\mathbf{v}'$ has been changed. According to this definition, if the gas-surface interaction is diffuse, then $\alpha(\varphi) = 1$ for any function $\varphi(\mathbf{v})$. That is why the diffuse reflection is also called the perfect accommodation.

The diffuse scattering kernel can be successfully used in many practical calculations. But some experimental data and corresponding results of numerical calculations based on the kernel (5) do not agree each with other. Rarefied gas flows through channels and tubes are one of the problems where the disagreement was observed. In the literature, there are many reliable numerical data on the rarefied gas flow between two parallel plates, see, e.g., [2–11] or through a cylindrical capillary, see, e.g., [2,12–16]. In these papers the mass flow rate was calculated assuming the diffuse reflection (5). The extensive list of the papers on this type of the flow can be found in the review [17]. However, for some noble gases, e.g., helium, neon, etc., experimental values of the mass flow rate through capillaries [18] are larger than those calculated in the above mentioned papers.

To eliminate this discrepancy the diffuse – specular Maxwell kernel

$$R_M(\mathbf{v}' \to \mathbf{v}) = (1-\varepsilon)\delta(\mathbf{v}' - \mathbf{v} + 2\mathbf{n}v_n) + \varepsilon \frac{m^2 v_n}{2\pi (kT_w)^2} \exp\left(-\frac{mv^2}{2kT_w}\right) \tag{8}$$

is usually used. Physically this means that the ε part of particles are reflected diffusely and the rest part $(1-\varepsilon)$ is reflected specularly. Substituting this kernel into the definition (7) it is easily shown that $\alpha(\varphi) = \varepsilon$ for any function $\varphi(\mathbf{v})$, i.e., the parameter ε is the accommodation coefficient of all molecular properties.

There are much data on the mass flow rate through channel and tube based on the Maxwell kernel (8), see e.g. [16,19,20]. The coefficient ε was calculated in the works [18,19,21] comparing experimental data on the mass flow rate with the corresponding theoretical results. It would be very attractive to carry out experiments for various gases and various surfaces with the object to tabulate the parameter ε for every pair gas-surface. Then, these tables could be used in engineering calculations in the same way as the tables of other properties (viscosity, thermal conductivity, etc.) are used nowadays.

However, calculations of the parameter ε from different experimental data, e.g., Poiseuille flow and the thermomolecular pressure difference (TPD), give quite different values. The values of ε can be also different for the free-molecular and hydrodynamic regimes. For instance, in the work [18] (their Table 5) the value $\varepsilon = 0.935$ is given for the pair helium-glass calculated on the basis of the free-molecular Poiseuille flow. In the same work [18] (their Table 4) it is reported the value $\varepsilon = 0.895$, obtained from the data on the slip coefficient for the same pair gas-surface. In the paper [19] one finds the value $\varepsilon = 0.68$ calculated from the data on the TPD also for the pair helium-glass.

One more contradiction is that, applying the Maxwell scattering kernel (8) to calculate the TPD in the free-molecular regime one obtains the exponent $\gamma = 0.5$ for any value of the accommodation coefficient ε, while the experimental data [22] show that a lower value of γ is possible.

So, the Maxwell kernel cannot correctly describe the gas-surface interaction. The main reason of such failure is the fact that the Maxwell kernel assumes all accommodation coefficients to be equal to ε, while in practice every property has its own accommodation coefficient. It became clear that one parameter for the scattering kernel is not enough, but it should have at least two parameters.

Almost thirty years ago Cercignani and Lampis [23] offered the following scattering kernel

$$R_{CL}(\mathbf{v}' \to \mathbf{v}) = \frac{m^2 v_n}{2\pi \alpha_n \alpha_t (2-\alpha_t)(kT_w)^2}$$

$$\times \exp\left\{-\frac{m[v_n^2+(1-\alpha_n)v_n'^2]}{2kT_w\alpha_n}-\frac{m[\mathbf{v}_t-(1-\alpha_t)\mathbf{v}_t']^2}{2kT_w\alpha_t(2-\alpha_t)}\right\}$$
$$\times I_0\left(\frac{\sqrt{1-\alpha_n}\,mv_nv_n'}{\alpha_nkT_w}\right), \tag{9}$$

where

$$I_0(x)=\frac{1}{2\pi}\int_0^{2\pi}\exp(x\cos\phi)\,\mathrm{d}\phi, \tag{10}$$

$\mathbf{v}_t$ is the two-dimensional vector of the tangential velocity. This kernel satisfies the normalization condition (3) and the reciprocity relation (4). Recently, the Cercignani–Lampis (CL) model has been extended to polyatomic gases [24].

The kernel (9) contains the two parameters α_t and α_n. To clarify the physical meaning of these parameters we substitute the kernel $R_{CL}(\mathbf{v}'\to\mathbf{v})$ into (7) and taking into account that

$$\int_{v_n>0} mv_{t_1}R_{CL}(\mathbf{v}'\to\mathbf{v})\,\mathrm{d}\mathbf{v} \;=\; (1-\alpha_t)mv_{t_1}', \tag{11}$$

$$\int_{v_n>0} \frac{mv_n^2}{2}R_{CL}(\mathbf{v}'\to\mathbf{v})\,\mathrm{d}\mathbf{v} \;=\; \alpha_nkT_w+(1-\alpha_n)\frac{mv_n'^2}{2}, \tag{12}$$

we obtain

$$\alpha(\varphi) \;=\; \alpha_t, \quad \text{if } \varphi(\mathbf{v})=mv_{t_1}, \tag{13}$$

$$\alpha(\varphi) \;=\; \alpha_n, \quad \text{if } \varphi(\mathbf{v})=\frac{mv_n^2}{2}, \tag{14}$$

where v_{t_1} is one tangential component of the two-dimensional vector $\mathbf{v}_t$. Thus, the first parameter α_t is the accommodation coefficient of the tangential momentum and the second one α_n is the accommodation coefficient of the kinetic energy due to the normal velocity v_n.

One can verify that if both accommodation coefficients are equal to unity the kernel (9) coincides with the diffuse kernel (4). In the other limit case $\alpha_t=0$ and $\alpha_n=0$ the kernel (9) becomes the specular one. Moreover, it includes the back scattering at $\alpha_t=2$ and $\alpha_n=0$, i.e., after a collision we have $\mathbf{v}=-\mathbf{v}'$.

Because of the complexity the CL kernel (9) has not yet been widely applied. In the papers [25,26] this model was used to calculate the Poiseuille flow and the thermal creep through a tube in the free-molecular regime. The authors of these works assumed the coefficients α_t and α_n to be close to unity. Their theoretical result based on the CL kernel gives the exponent of the TPD lower than 0.5. This fact shows the kernel (9) provides a more physical description of the non-diffuse gas-surface interaction than the Maxwell boundary condition (8). To confirm this we have to apply the kernel (9) to numerical calculations of rarefied gas flows in the wide range of the Knudsen number varying the coefficients α_t and α_n in large intervals. A comparison of these results with experimental data will allow us to calculate the accommodation coefficients α_t and α_n. Indeed, to perform a rigorous verification of the applicability of the kernel (9) or any other one a set of different experiments, e.g., Poiseuille flow, Couette flow, thermal creep, transversal heat flux between two plates, etc., should be carried out with the same gas and the same surface.

The aim of the present paper is to calculate the Poiseuille flow, the thermal creep, the mechanocaloric flux and the heat flux along two parallel plates as a function of the accommodation coefficients α_t and α_n in the wide range of the Knudsen number.

2. Input equation

Consider a monoatomic rarefied gas confined between two infinite plates fixed at $y'=\pm a/2$. There are longitudinal gradients of the pressure P and temperature T

$$\upsilon=\frac{a}{P}\frac{\mathrm{d}P}{\mathrm{d}x'}, \qquad \tau=\frac{a}{T}\frac{\mathrm{d}T}{\mathrm{d}x'}, \tag{15}$$

which cause the gas flow along the plates. We are going to calculate the mass flow rate and the heat flux through a cross section of the channel.

For further derivations it is convenient to introduce the following dimensionless quantities

$$x=\frac{x'}{a},\qquad y=\frac{y'}{a},\tag{16}$$

$$\mathbf{c}=\beta\mathbf{v},\qquad u=\beta u'_x,\qquad q=\frac{\beta}{P}q'_x,\qquad \beta=\left(\frac{m}{2kT}\right)^{1/2},\tag{17}$$

where u'_x is the longitudinal component of the bulk velocity, q'_x is the longitudinal component of the heat flux. We assume the bulk velocity and the heat flux vector have an x component only.

Since the pressure and temperature gradients are small, the distribution function $f(\mathbf{r},\mathbf{c})$ can be linearized as

$$f(\mathbf{r},\mathbf{c})=f^0\left[1+h(y,\mathbf{c})+\upsilon x+\left(c^2-\frac{5}{2}\right)\tau x\right],\tag{18}$$

$$f^0=n_0\left(\frac{m}{2\pi kT_0}\right)^{3/2}\exp\left(-\frac{mv^2}{2kT_0}\right),$$

where n_0 and T_0 are equilibrium number density and temperature, respectively.

As was shown in the review [17] the S model of the Boltzmann equation (1) proposed by Shakhov [27] provides reliable numerical results for non-isothermal rarefied gas flows with modest computational efforts. The advantage of this model against the widely used the BGK equation [28] is that it gives the correct value of the Prandtl number. Physically it means that the S model provides the correct expressions of the viscosity and heat conductivity simultaneously, while the BGK model provides correctly only one of these coefficients. That is why in the present work the S model equation is applied, which in the linearized form becomes

$$c_y\frac{\partial h}{\partial y}=\delta\left[2uc_x+\frac{4}{15}qc_x\left(c^2-\frac{5}{2}\right)-h\right]-c_x\left[\upsilon+\tau\left(c^2-\frac{5}{2}\right)\right],\tag{19}$$

where

$$u(y)=\frac{1}{\pi^{3/2}}\int\exp\left(-c^2\right)h(y,\mathbf{c})c_x\,\mathrm{d}\mathbf{c},\tag{20}$$

$$q(y)=\frac{1}{\pi^{3/2}}\int\exp\left(-c^2\right)h(y,\mathbf{c})c_x\left(c^2-\frac{5}{2}\right)\mathrm{d}\mathbf{c},\tag{21}$$

δ is the rarefaction parameter defined as

$$\delta=\frac{\sqrt{\pi}}{2}\frac{a}{\lambda},\tag{22}$$

λ is the mean free path related to the stress viscosity μ as

$$\lambda=\frac{\mu}{P}\left(\frac{\pi kT}{2m}\right)^{1/2}.\tag{23}$$

Since Eq. (19) is linear, its solution h and the moments u and q can be decomposed into two parts as

$$h=h_P\upsilon+h_T\tau,\tag{24}$$

$$u=u_P\upsilon+u_T\tau,\tag{25}$$

$$q=q_P\upsilon+q_T\tau.\tag{26}$$

Then, we introduce the two mass flow rates

$$G_P=-2\int_{-1/2}^{1/2}u_P(y)\,\mathrm{d}y,\qquad G_T=2\int_{-1/2}^{1/2}u_T(y)\,\mathrm{d}y,\tag{27}$$

and the two heat fluxes

$$Q_P=2\int_{-1/2}^{1/2}q_P(y)\,\mathrm{d}y,\qquad Q_T=-2\int_{-1/2}^{1/2}q_T(y)\,\mathrm{d}y.\tag{28}$$

The coefficient G_P is called the Poiseuille flow, Q_T is the ordinary heat flux, the thermal creep G_T and the mechanocaloric flux Q_P describe the cross effects. In the papers [29–31] it was proved that for any scattering kernel satisfying Eq. (4) the cross effects G_T and Q_P obey the Onsager relation

$$G_T = Q_P \tag{29}$$

in the whole range of the rarefaction parameter δ. This relation can be use as an additional accuracy criterion or to reduce the computational efforts.

To eliminate the variables c_x and c_z two functions are introduced as

$$\varphi_j(y,c_y) = \frac{1}{\pi}\iint \exp\bigl(-c_x^2-c_z^2\bigr)h_j(y,\mathbf{c})c_x\,\mathrm{d}c_x\,\mathrm{d}c_z, \tag{30}$$

$$\psi_j(y,c_y) = \frac{1}{\pi}\iint \exp\bigl(-c_x^2-c_z^2\bigr)h_j(y,\mathbf{c})c_x\bigl(c_x^2+c_z^2-2\bigr)\mathrm{d}c_x\,\mathrm{d}c_z, \tag{31}$$

where $j = P, T$. Multiplying Eq. (19) by

$$\pi^{-1}\exp\bigl(-c_x^2-c_z^2\bigr)c_x$$

and integrating it with respect to c_x and c_z one obtains

$$c_y\frac{\partial\varphi_j}{\partial y} = \delta\left[u_j+\frac{2}{15}q_j\left(c_y^2-\frac{1}{2}\right)-\varphi_j\right]-S_j^{\varphi}, \tag{32}$$

where

$$S_P^{\varphi}=\frac{1}{2},\qquad S_T^{\varphi}=\frac{1}{2}\left(c_y^2-\frac{1}{2}\right). \tag{33}$$

Then, multiplying Eq. (19) by

$$\pi^{-1}\exp\bigl(-c_x^2-c_z^2\bigr)c_x\bigl(c_x^2+c_z^2-2\bigr)$$

the following equations for ψ_P and ψ_T are obtained

$$c_y\frac{\partial\psi_j}{\partial y} = \delta\left(\frac{4}{15}q_j-\psi_j\right)-S_j^{\psi},\quad j=P,T, \tag{34}$$

where

$$S_P^{\psi}=0,\qquad S_T^{\psi}=1. \tag{35}$$

The moments are calculated via φ_j and ψ_j as

$$u_j(y) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\exp\bigl(-c_y^2\bigr)\varphi_j(y,c_y)\,\mathrm{d}c_y, \tag{36}$$

$$q_j(y) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\exp\bigl(-c_y^2\bigr)\left[\varphi_j(y,c_y)\left(c_y^2-\frac{1}{2}\right)+\psi_j(y,c_y)\right]\mathrm{d}c_y, \tag{37}$$

where $j = P, T$. One can see that Eqs. (32) and (34) are coupled via the heat flux q_j.

3. Linearized boundary conditions

In terms of the dimensionless velocity $\mathbf{c}$ introduced in (17) the CL kernel can be written as

$$R_{CL}(\mathbf{c}'\to\mathbf{c}) = R_n(c_n'\to c_n)R_x(c_x'\to c_x)R_z(c_z'\to c_z), \tag{38}$$

where

$$R_n(c_n'\to c_n) = \frac{2c_n}{\alpha_n}\exp\left[-\frac{c_n^2+(1-\alpha_n)c_n'^2}{\alpha_n}\right]I_0\left(\frac{2\sqrt{1-\alpha_n}\,c_nc_n'}{\alpha_n}\right), \tag{39}$$

$$R_i(c_i'\to c_i) = \frac{1}{[\pi\alpha_t(2-\alpha_t)]^{1/2}}\exp\left\{-\frac{[c_i-(1-\alpha_t)c_i']^2}{\alpha_t(2-\alpha_t)}\right\},\quad i=x,z. \tag{40}$$

Note that every component of the kernel is normalized as

$$\int\limits_{c_n>0} R_n(c_n' \to c_n)\,\mathrm{d}c_n = 1, \tag{41}$$

$$\int\limits_{-\infty}^{+\infty} R_i(c_i' \to c_i)\,\mathrm{d}c_i = 1. \tag{42}$$

For the problem in question $c_n = c_y$ at $y = -1/2$ and $c_n = -c_y$ at $y = 1/2$. Moreover, it has been assumed the surface temperature T_w to be equal to the gas temperature in every channel section. Substituting (18) into (2) with the help of (3) and (4) we relate the perturbation of incident particles h^- to the perturbation of reflected ones h^+ as

$$h^+ = \hat{A}_n \hat{A}_x \hat{A}_z h^-, \tag{43}$$

where the following scattering operators have been introduced

$$\hat{A}_n \phi = \frac{1}{c_n} \exp\bigl(c_n^2\bigr) \int\limits_{c_n'<0} |c_n'| \exp\bigl(-c_n'^2\bigr) R_n(c_n' \to c_n)\phi(c_n')\,\mathrm{d}c_n', \tag{44}$$

$$\hat{A}_i \phi = \exp\bigl(c_i^2\bigr) \int\limits_{-\infty}^{\infty} \exp\bigl(-c_i'^2\bigr) R_i(c_i' \to c_i)\phi(c_i')\,\mathrm{d}c_i', \quad i = x, z. \tag{45}$$

Here, ϕ is an arbitrary velocity function. It can be shown that the operators $\hat{A}_x$ and $\hat{A}_z$ satisfy the relations

$$\hat{A}_x c_x = (1-\alpha_t)c_x, \tag{46}$$

$$\hat{A}_x c_x^2 = (1-\alpha_t)^2 c_x^2 + \frac{1}{2}\alpha_t(2-\alpha_t), \tag{47}$$

$$\hat{A}_x c_x^3 = (1-\alpha_t)^3 c_x^3 + \frac{3}{2} c_x \alpha_t (2-\alpha_t)(1-\alpha_t), \tag{48}$$

$$\hat{A}_x \hat{A}_z\bigl(c_x^2 + c_z^2 - 2\bigr) = (1-\alpha_t)^3\bigl(c_x^2 + c_z^2 - 2\bigr). \tag{49}$$

The perturbation functions h_P and h_T can be expressed via the corresponding functions $\varphi_j(c_y)$ and $\psi_j(c_y)$ as

$$h_j(y, \mathbf{c}) = 2\varphi_j(y, c_y)c_x + \psi_j(y, c_y)c_x\bigl(c_x^2 + c_z^2 - 2\bigr), \quad j = P, T. \tag{50}$$

Substituting (50) into (43) and applying Eqs. (46)–(49) we obtain the boundary condition for the functions $\varphi_j(c_y)$ and $\psi_j(c_y)$

$$\varphi_j^+ = (1-\alpha_t)\hat{A}_n \varphi_j^-, \qquad \psi_j^+ = (1-\alpha_t)^3 \hat{A}_n \psi_j^-. \tag{51}$$

From the boundary conditions in this form we conclude that if $\alpha_t = 1$, then the coefficients G_P, G_T, Q_P and Q_T do not depend on the energy accommodation coefficient α_n. The same result was obtained in the works [25,26]. Note that this it is a peculiarity of the longitudinal rarefied gas flow. If one considers a transversal heat flux between two parallel plates having different temperature one finds a dependence of the heat flux on α_n at $\alpha_t = 1$.

4. Results and discussions

The kinetic equations (32) and (34) with the boundary condition (51) were solved by the optimized discrete velocity method [9] with the numerical error less than 0.1%. The numerical accuracy was estimated by comparing the numerical values of the coefficients G_P, G_T, Q_P and Q_T for different grid parameters. An analysis of the numerical data showed that the Onsager relation (29) is fulfilled within the numerical accuracy. The results of the calculations are presented in Tables 1 to 3. It should be note that in case of the diffuse reflection the present results are in a good agreement with the previous works [10,11].

One can see that the Poiseuille flow G_P given in Table 1 significantly depends on the momentum accommodation coefficient α_t in the whole range of the rarefaction parameter δ considered here. It always decreases by increasing the accommodation coefficient α_t. The dependence of the Poiseuille flow G_P on the energy accommodation coefficient α_n is very weak. It is interesting that in the interval $0.01 \leqslant \delta \leqslant 2.0$ this dependence decreases by increasing the rarefaction parameter and

Table 1
Poiseuille flow G_P versus δ, α_t and α_n

		G_P			
δ	α_t	$\alpha_n = 0.25$	0.5	0.75	1.0
0.01	0.25	9.230	8.699	8.480	8.369
	0.5	5.262	5.014	4.893	4.824
	0.75	3.822	3.726	3.675	3.642
	1.0	3.052	3.052	3.052	3.052
0.1	0.25	7.918	7.596	7.441	7.356
	0.5	4.109	3.952	3.865	3.812
	0.75	2.754	2.693	2.655	2.630
	1.0	2.039	2.039	2.039	2.039
0.2	0.25	7.536	7.319	7.202	7.134
	0.5	3.808	3.699	3.633	3.590
	0.75	2.501	2.457	2.429	2.408
	1.0	1.817	1.817	1.817	1.817
1.0	0.25	6.944	6.913	6.889	6.871
	0.5	3.370	3.352	3.338	3.326
	0.75	2.164	2.157	2.150	2.144
	1.0	1.554	1.554	1.554	1.554
2.0	0.25	6.940	6.963	6.932	6.928
	0.5	3.391	3.389	3.386	3.384
	0.75	2.205	2.204	2.203	2.202
	1.0	1.611	1.611	1.611	1.611
3.5	0.25	7.110	7.110	7.109	7.109
	0.5	3.566	3.565	3.565	3.565
	0.75	2.384	2.383	2.383	2.383
	1.0	1.792	1.792	1.792	1.792
10.0	0.25	8.125	8.114	8.104	8.095
	0.5	4.572	4.564	4.557	4.551
	0.75	3.380	3.377	3.373	3.370
	1.0	2.780	2.780	2.780	2.780
20.0	0.25	9.769	9.751	9.734	9.719
	0.5	6.210	6.198	6.186	6.176
	0.75	5.012	5.006	5.001	4.996
	1.0	4.405	4.405	4.405	4.405
100.0	0.25	23.06	23.04	23.02	23.00
	0.5	19.51	19.49	19.47	19.46
	0.75	18.31	18.30	18.29	18.29
	1.0	17.70	17.70	17.70	17.70

at $\delta = 3.5$ the Poiseuille flow does not depend on the energy accommodation coefficient α_n. Then, by increasing the rarefaction parameter δ this dependence appears again and begins to vanish near the hydrodynamic regime ($\delta \to \infty$).

The thermal creep G_T shown in Table 2 also depends on the momentum accommodation coefficient α_t. However, near the free molecular regime the coefficient G_T decreases, while near the hydrodynamic regime it increases, by increasing the accommodation coefficient α_t. The dependence of the thermal creep G_T on the energy accommodation coefficient α_n is similar to that on the coefficient α_t, i.e., near the free molecular regime it decreases and near the hydrodynamic regime it increases by increasing the accommodation coefficient α_n. For the rarefaction parameter $\delta = 3.5$ the thermal creep G_T is practically constant.

Since the mechanocaloric flux Q_P is equal to G_T, it is not commented here.

It is rather surprising result that the heat flux Q_T presented in Table 3 weakly depends on the energy accommodation coefficient α_n. The dependence practically vanishes near the hydrodynamic regime. A significant dependence of the heat flux Q_T on the momentum accommodation coefficient α_t is observed only near the free molecular regime.

Table 2
Thermal creep G_T and mechanocaloric flux Q_P versus δ, α_t and α_n

		$G_T = Q_P$			
δ	α_t	$\alpha_n = 0.25$	0.5	0.75	1
0.01	0.25	2.005	1.560	1.360	1.251
	0.5	1.621	1.423	1.315	1.248
	0.75	1.397	1.324	1.279	1.247
	1.0	1.247	1.247	1.247	1.247
0.1	0.25	1.308	1.001	0.8420	0.7496
	0.5	1.018	0.8780	0.7928	0.7362
	0.75	0.8484	0.7954	0.7596	0.7331
	1.0	0.7327	0.7327	0.7327	0.7327
0.2	0.25	1.064	0.8374	0.7084	0.6288
	0.5	0.8369	0.7306	0.6608	0.6121
	0.75	0.7015	0.6604	0.6308	0.6080
	1.0	0.6074	0.6075	0.6075	0.6075
1.0	0.25	0.5003	0.4508	0.4122	0.3819
	0.5	0.4377	0.4110	0.3886	0.3698
	0.75	0.3963	0.3851	0.3750	0.3660
	1.0	0.3654	0.3655	0.3655	0.3655
2.0	0.25	0.3168	0.3038	0.2926	0.2828
	0.5	0.2969	0.2895	0.2828	0.2765
	0.75	0.2838	0.2806	0.2774	0.2744
	1.0	0.2741	0.2741	0.2741	0.2741
3.5	0.25	0.2084	0.2082	0.2083	0.2084
	0.5	0.2055	0.2056	0.2057	0.2058
	0.75	0.2046	0.2047	0.2048	0.2048
	1.0	0.2046	0.2047	0.2047	0.2047
10.0	0.25	0.08592	0.08997	0.09400	0.09794
	0.5	0.09018	0.09290	0.09556	0.09819
	0.75	0.09419	0.09554	0.09684	0.09814
	1.0	0.09808	0.09813	0.09813	0.09813
20.0	0.25	0.04519	0.04794	0.05066	0.05333
	0.5	0.04830	0.05019	0.05200	0.05380
	0.75	0.05111	0.05206	0.05294	0.05383
	1.0	0.05378	0.05383	0.05383	0.05383
100.0	0.25	0.00938	0.01008	0.01077	0.01144
	0.5	0.01015	0.01064	0.01108	0.01152
	0.75	0.01085	0.01110	0.01132	0.01154
	1.0	0.01151	0.01154	0.01154	0.01154

As has been shown above the parameter ε in the Maxwell kernel is the accommodation coefficient of every molecular property. So, one may expect that assuming

$$\varepsilon = \alpha_t = \alpha_n, \tag{52}$$

the Maxwell kernel and the CL kernel will provide the results close to each other. In Table 4 a comparison of numerical results obtained for both kernels under the condition (52) is performed. One can see that the Poiseuille flow G_P weakly depends on the scattering kernel in the transition ($\delta = 1$) and near hydrodynamic ($\delta = 10$) regimes, while near the free molecular regime ($\delta = 0.1$) the dependence is significant. The thermal creep G_T, the mechanocaloric flux Q_T, and the heat flux Q_T are more sensible to the scattering kernel then the Poiseuille flow G_P. The difference of these coefficient due to the scattering kernel decreases in the hydrodynamic regime.

Table 3
Heat flux Q_T versus δ, α_t and α_n

		Q_T			
δ	α_t	$\alpha_n = 0.25$	0.5	0.75	1
0.01	0.25	10.53	9.776	9.406	9.191
	0.5	7.710	7.460	7.315	7.218
	0.75	6.939	6.869	6.823	6.788
	1.0	6.734	6.734	6.734	6.734
0.1	0.25	6.761	6.299	6.046	5.888
	0.5	4.801	4.628	4.516	4.437
	0.75	4.220	4.167	4.128	4.098
	1.0	4.054	4.054	4.054	4.054
0.2	0.25	5.397	5.072	4.877	4.750
	0.5	3.918	3.785	3.693	3.627
	0.75	3.452	3.409	3.375	3.348
	1.0	3.312	3.312	3.312	3.312
1.0	0.25	2.384	2.301	2.235	2.184
	0.5	1.976	1.934	1.897	1.866
	0.75	1.815	1.797	1.782	1.767
	1.0	1.754	1.754	1.754	1.754
2.0	0.25	1.453	1.419	1.389	1.362
	0.5	1.291	1.272	1.254	1.238
	0.75	1.220	1.212	1.203	1.195
	1.0	1.189	1.189	1.189	1.189
3.5	0.25	0.9212	0.9068	0.8933	0.8808
	0.5	0.8567	0.8481	0.8398	0.8318
	0.75	0.8263	0.8224	0.8184	0.8145
	1.0	0.8120	0.8121	0.8121	0.8121
10.0	0.25	0.3558	0.3537	0.3517	0.3498
	0.5	0.3471	0.3458	0.3445	0.3433
	0.75	0.3428	0.3422	0.3416	0.3409
	1.0	0.3406	0.3406	0.3406	0.3406
20.0	0.25	0.1827	0.1822	0.1817	0.1812
	0.5	0.1805	0.1802	0.1799	0.1796
	0.75	0.1794	0.1793	0.1791	0.1790
	1.0	0.1789	0.1789	0.1789	0.1789
100.0	0.25	0.03731	0.03728	0.03726	0.03724
	0.5	0.03722	0.03721	0.03719	0.03718
	0.75	0.03717	0.03717	0.03716	0.03716
	1.0	0.03715	0.03715	0.03715	0.03715

5. Conclusions

The Poiseuille flow, the thermal creep, the mechanocaloric flux and the heat flux have been calculated as function of the rarefaction parameter δ and of the accommodation coefficients α_t and α_n. The calculations have been carried out on the basis of the S model kinetic equation with the CL model of the gas-surface interaction. Unfortunately, it is impossible to compare the present numerical results with experimental data and to calculate the accommodation coefficients α_t and α_n for some specific gases and surfaces, because in open literature the graphic presentation of experimental data is used, see, e.g., the work [18]. Such presentation does not allow to perform an accurate comparison of theoretical and experimental data. To realize this task it is necessary to perform high accuracy experiments on both isothermal and non-isothermal gas flows through a plane channel and to tabulate the experimental data.

Other types of rarefied gas flows will be presented in forthcoming papers.

Table 4
Comparison of numerical results based on the Maxwell (M) and Cercignani–Lampis (CL) kernels at $\varepsilon = \alpha_t = \alpha_n$

		G_P		$G_T = Q_P$		Q_T	
δ	ε	M	CL	M	CL	M	CL
0.1	0.25	8.980	7.918	2.283	1.308	12.90	6.761
	0.5	4.580	3.952	1.401	0.8780	7.977	4.628
	0.75	2.936	2.655	0.9858	0.7596	5.559	4.128
1.0	0.25	6.987	6.944	0.5911	0.5003	2.988	2.384
	0.5	3.392	3.352	0.4903	0.4110	2.464	1.934
	0.75	2.174	2.150	0.4191	0.3750	2.068	1.782
10.0	0.25	8.141	8.125	0.08139	0.08592	0.3657	0.3558
	0.5	4.582	4.564	0.08741	0.09290	0.3569	0.3458
	0.75	3.385	3.373	0.09295	0.09684	0.3485	0.3416

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